Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet answers for 2021-11-12

If you would like clarification on any problems, feel free to ask me in person. (Do let me know if you catch any mistakes!)

Answers to computations

Problem 1. It turns out $\mathbf{r}_u \times \mathbf{r}_v$ is the correct answer for all three of these problems (by complete coincidence; I didn't solve them in advance).

(a) For a parametrization of the form $\langle x, y, f(x,y)\rangle$, the rule is that $\mathbf{r}_x \times \mathbf{r}_y$ gives the upwards orientation (i.e. it will have positive z-component, namely 1).

(b) One way to do it is to draw the grid curves and use RHR. Alternatively, let's just compute $\mathbf{r}_u \times \mathbf{r}_v$ and see if it's the right answer:

$$\mathbf{r}_u \times \mathbf{r}_v = \det\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3\sin u & 3\cos u & 0 \\ 0 & 0 & 1 \end{bmatrix} = \langle 3\cos u, 3\sin u, 0\rangle.$$

If we plug in $u = 0, v = 0$, this gives the point $(3,0,0)$ on the cylinder, and the normal vector evaluates to $\langle 3,0,0\rangle$ as well. This points outwards, as desired, so we made the right choice. (Otherwise we would take the negative of the normal we computed.)

(c) Same approach works.

Problem 2. The given information is $\iint_S \mathrm{d}S = 4\pi R^2$. The integral we are asked to compute is

$$\begin{aligned}\iint_S \langle x,y,z\rangle \cdot \mathbf{n}\,\mathrm{d}S &= \iint_S \langle x,y,z\rangle \cdot \frac{\langle 2x,2y,2z\rangle}{2\sqrt{x^2+y^2+z^2}}\,\mathrm{d}S \\ &= \iint_S \sqrt{x^2+y^2+z^2}\,\mathrm{d}S \\ &= \iint_S R\,\mathrm{d}S = \boxed{4\pi R^3}.\end{aligned}$$

Here we have used that $\nabla(x^2+y^2+z^2)$ produces a normal vector for the sphere, but then we have to divide by its magnitude to obtain a unit normal $\mathbf{n}$. Note that this does point in the desired direction (outwards).